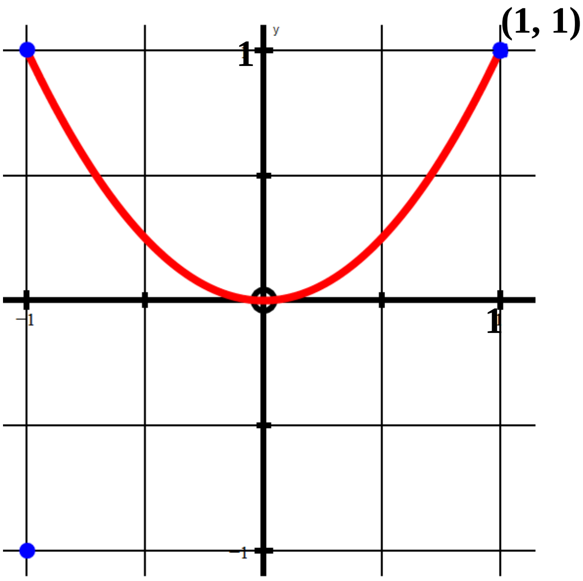
PROJECT: **<u>THE STRANGE BEHAVIOUR OF THE GRAPH $y = x^n$</u>**

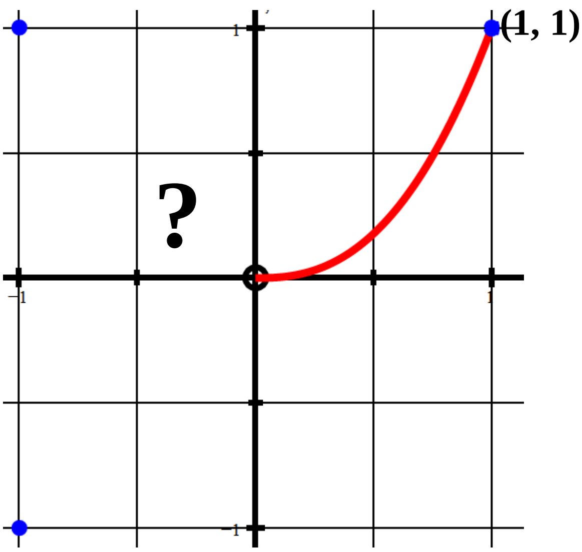
I found this topic to be absolutely fascinating!
I was "playing around" with my graphing program looking at graphs of the form **y = x^b** where **b** is not just a whole number.



This is the graph of $\mathbf{y = x^2}$

When I started increasing the power **b** in steps of **0.1** a **strange thing happened**.

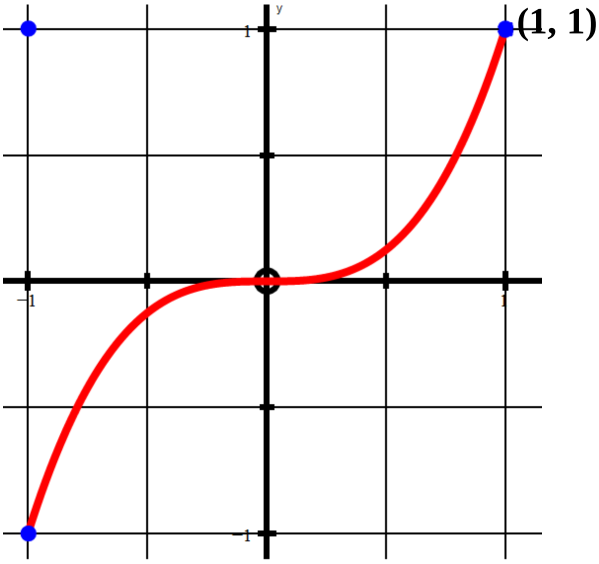
The **left hand** side of the graph just **disappeared!**



For example, this is the graph of $\mathbf{y = x^{2.5}}$

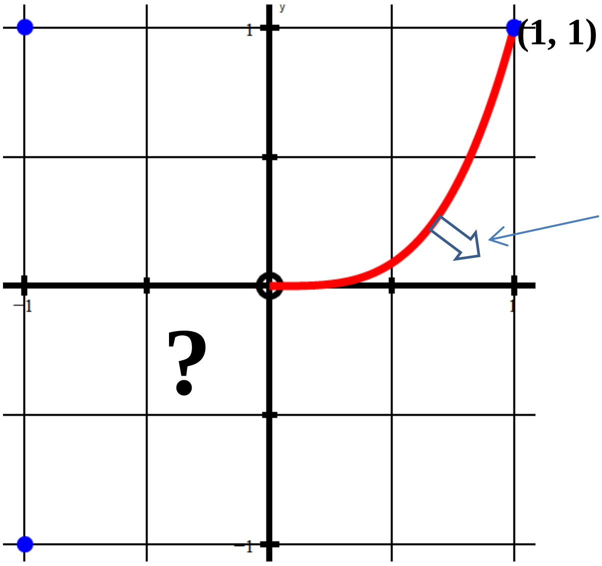
The right hand side of the graph is becoming more curved as **b** increases.

For all values of **b** up to **2.9** there is **no left hand side** to the graph!



Suddenly, when **b** becomes **3,** the left hand side **reappears again** and we have the graph of

$$\mathbf{y = x^3}$$

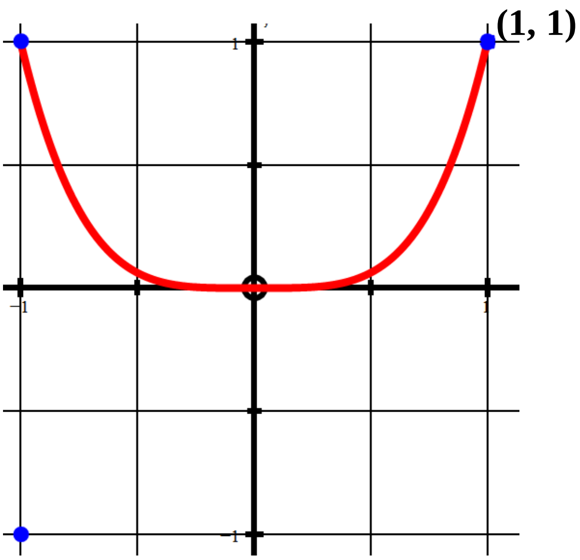


… but as soon as **b** increases to **3.1**, the left hand side disappears again but the right hand side continues to become more curved.

This curve is $\mathbf{y = x^{3.5}}$

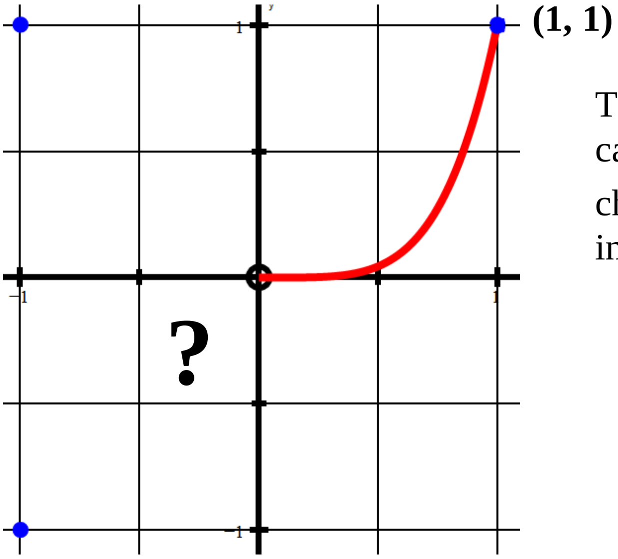
As **b** is increasing the right hand side is changing into

$$\mathbf{y = x^4}$$

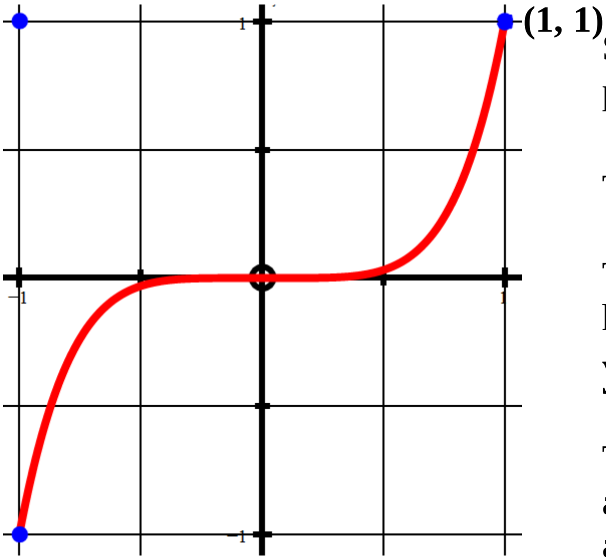


This is $\mathbf{y = x^4}$ and the left hand side has reappeared again.

Then as **b** increases further, the left hand side disappears again!



This graph is $\mathbf{y = x^{4.5}}$ and we can see the right hand side changing into $\mathbf{y = x^5}$ as **b** increases to **5.**



So I have slowly increased the power **b** from **2** to **5.**

This graph is $\mathbf{y = x^5}$

The right hand side of the graph has gradually changed from $\mathbf{y = x^2}$ to $\mathbf{y = x^5}$

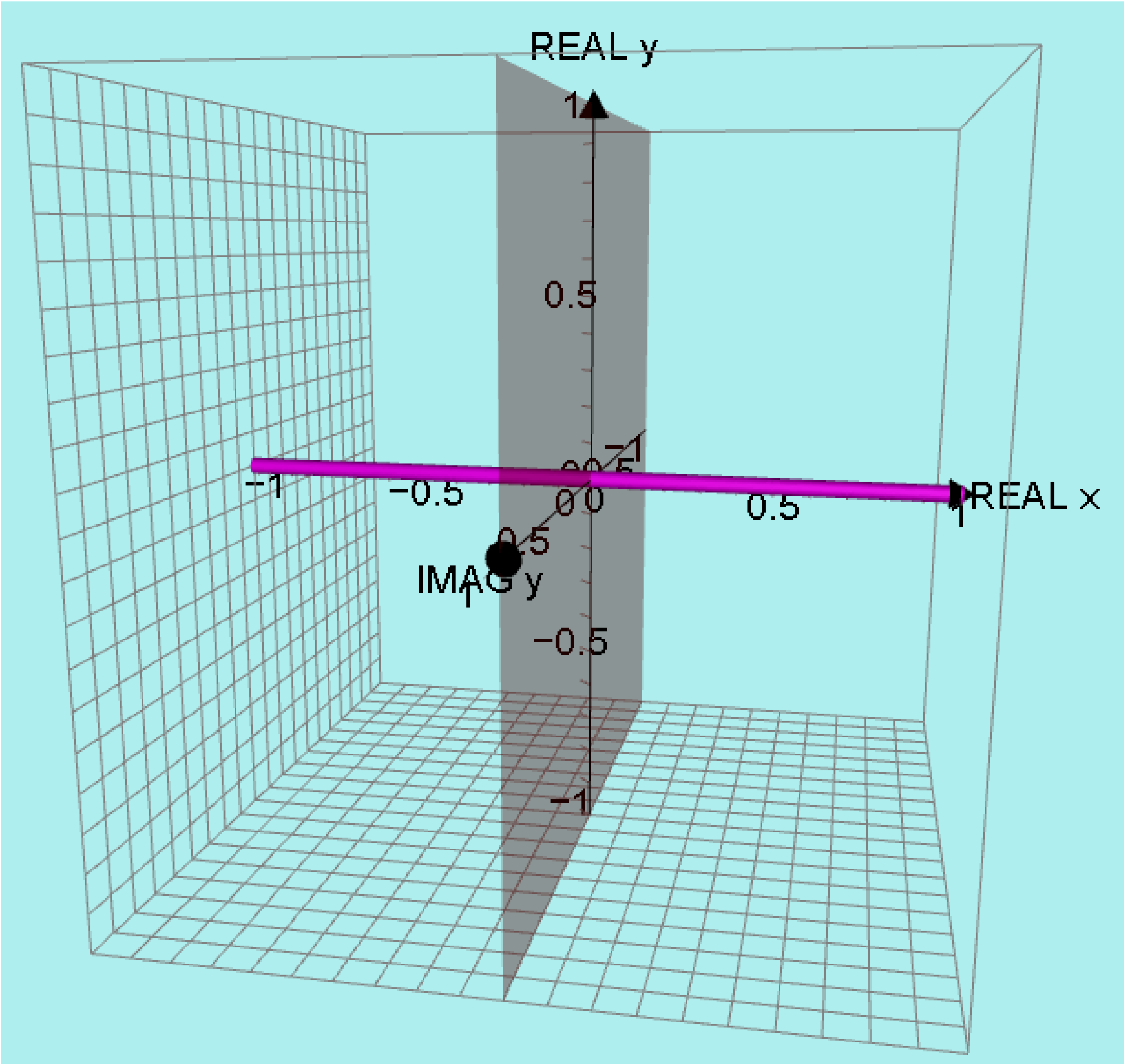
The left hand side has only appeared each time when **b** was a whole number.

The big question is: **"Where does the left hand side of the graph go to?"**

If I consider $\mathbf{y = x^{1.5}}$ and let $\mathbf{x = -1}$, I find that $\mathbf{(-1)^{1.5} = -\mathit{i}}$
Similarly $\mathbf{(-1)^{1.25} = -0.707 - 0.707i}$
and $\mathbf{(-1)^{1.75} = +0.707 - 0.707i}$

This means that although the **x** values are just the **real numbers** on the **x axis**, the **y values** have some **imaginary parts**!

This means we need to create a **complex y plane** instead of just a **y axis**. Like this…



Then to my utter delight, I found that the left hand sides of the graphs do not disappear at all!

<u>THEORY TO PRODUCE THE PURPLE PHANTOMS.</u>

The way to work out these results such as $\mathbf{(-1)^{1.25} = -0.707 - 0.707i}$ without a calculator is as follows…
Firstly change **(–1) to polar form = (+1)cis(180) or in rads cis(π)**
Now we use De Moivre's theorem:

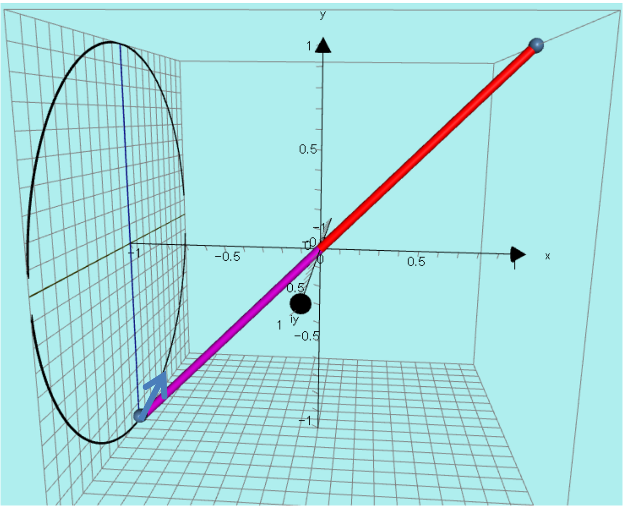
$$(-1)^{1.25} = (+1)^{1.25}\text{cis}(1.25\times180) = \cos(225) + i\sin(225) = -0.707 - 0.707i$$

So if the graph has the equation $y = x^n$ **and we are using just negative x values which have an argument of** 180^0 **or π rad.**
(I would be using rads in the Autograph graphing program.)
So following the above method…

$$y = x^n = |x|^n \times \text{cis}(n\pi)$$

The parametric equations for Autograph for **just the left hand side of the graphs are** $x = t,\ y = |t|^n \times \cos(n\pi),\ z = |t|^n \times \sin(n\pi)$

(NB the ordinary right hand side $y = x^n$ is written as $x = t,\ y = t^n,\ z = 0$)

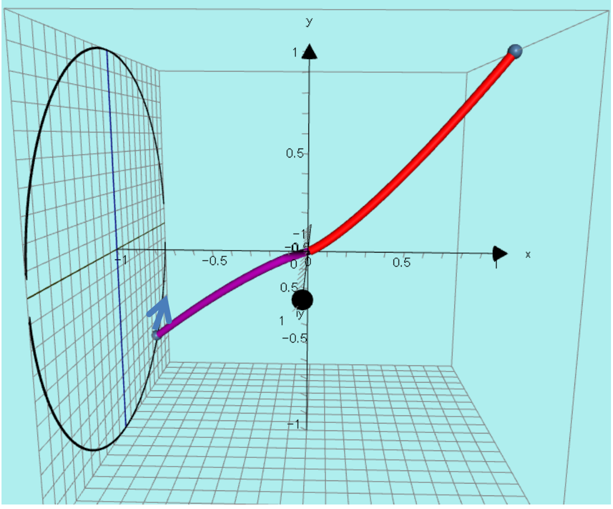


Starting with the graph of

$$y = x^1$$

I have coloured the right hand side **RED** which stays in the **x, y** plane.

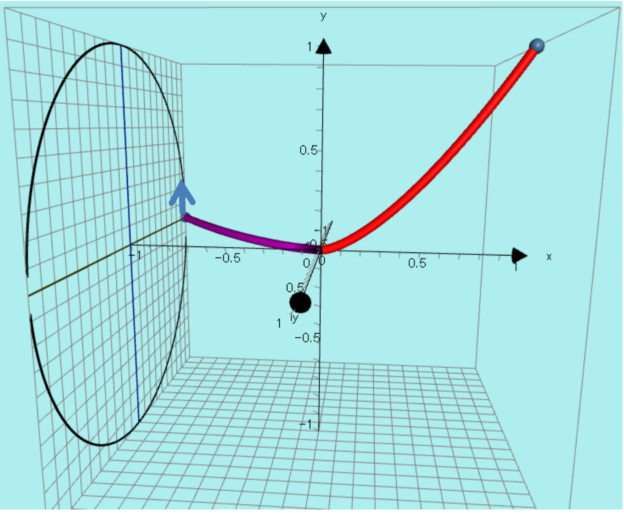
The left hand side is coloured **PURPLE**.



I have increased the power to

$$y = x^{1.25}$$

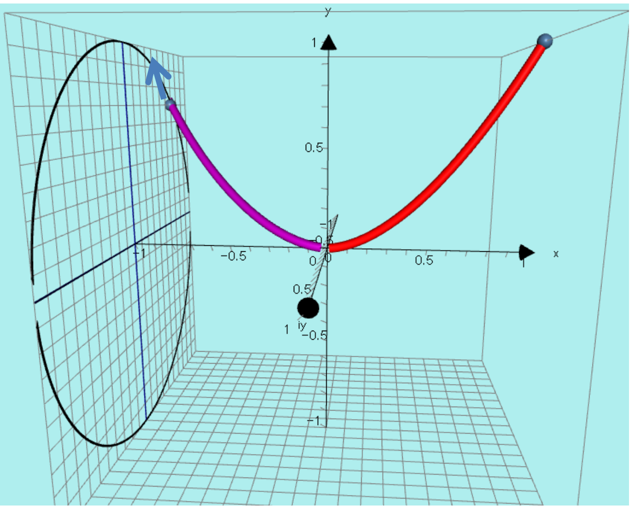
…and we see that the left hand part of the curve has **rotated** in an **anticlockwise** direction and is no longer in the x, y plane!



When the power is increased to

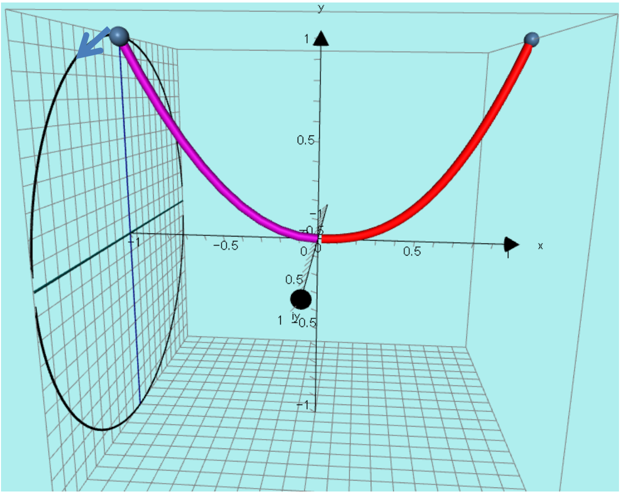
$$y = x^{1.5}$$

…we see that the purple section has rotated so that it is in a plane **perpendicular** to the **x, y** plane.



The rotation continues and the equation is now…

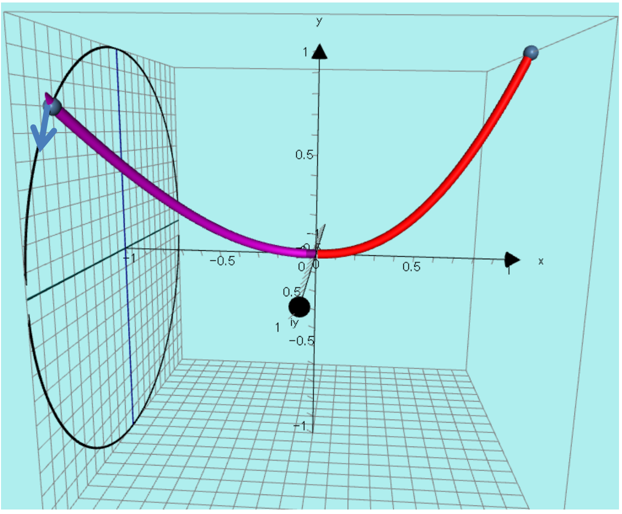
$$y = x^{1.75}$$



When the power is **2**, the purple curve is back in the **x, y** plane and the equation is...

$$\mathbf{y = x^2}$$

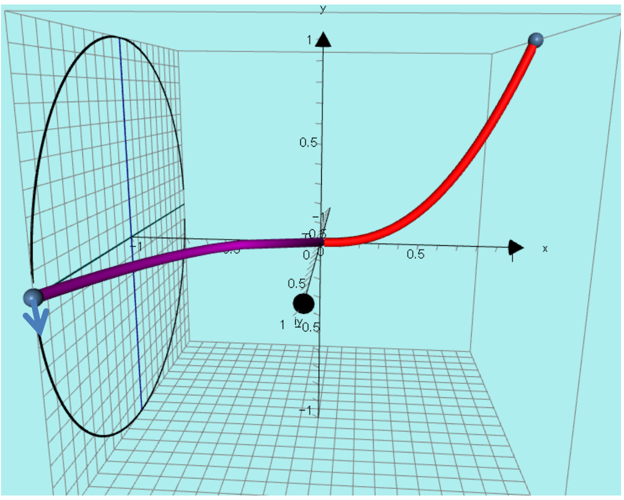
(This process continues for the other x values between the whole numbers)



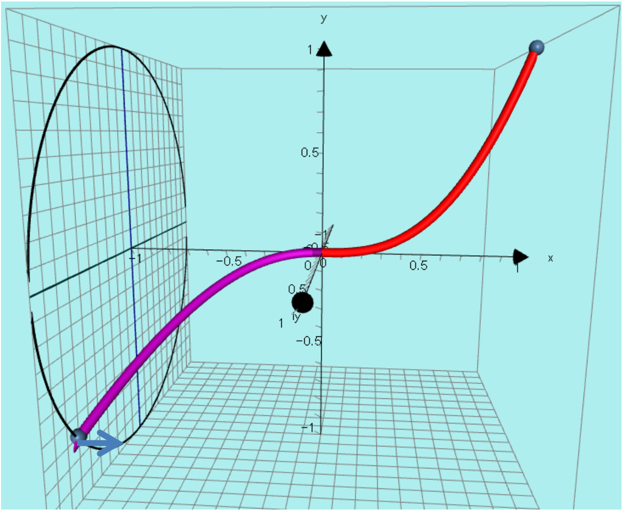
Increasing the power further, the curve continues rotating…

…the equation here is:

$$\mathbf{y = x^{2.25}}$$

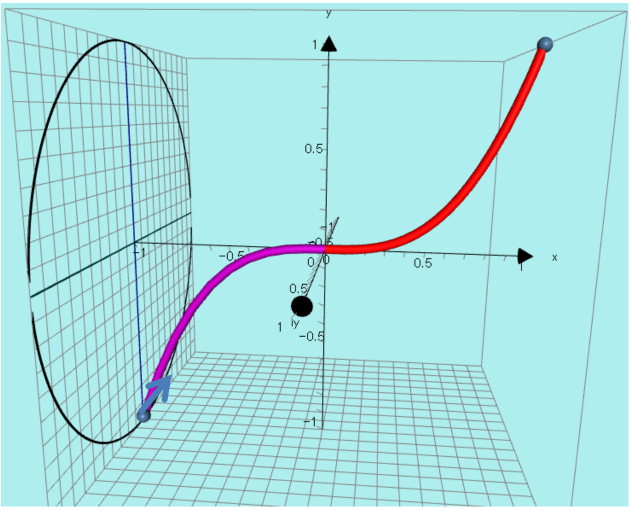


When the equation is $\mathbf{y = x^{2.5}}$ the purple section of the curve is again in a plane perpendicular to the **x, y** plane.



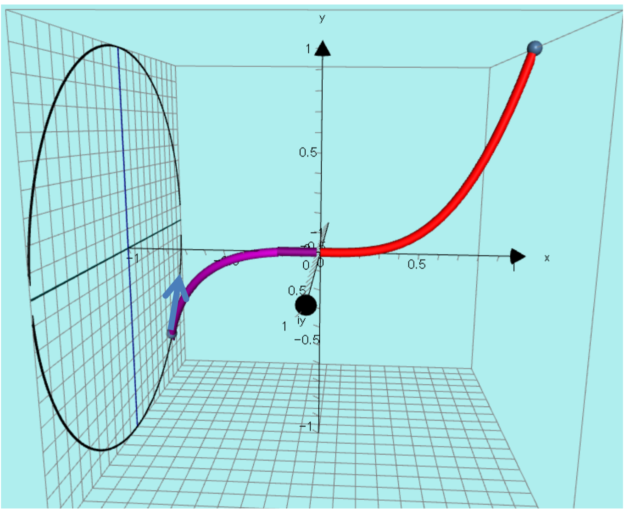
…the equation here is:

$$y = x^{2.75}$$



When the power is **3**, the purple curve is back in the **x, y** plane and the equation is...

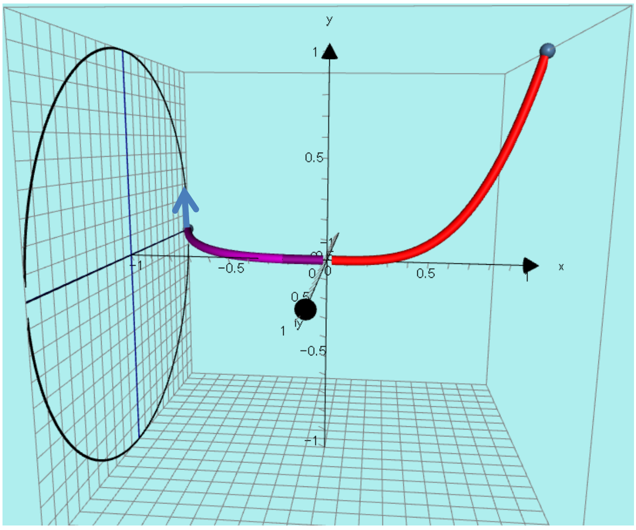
$$y = x^{3}$$



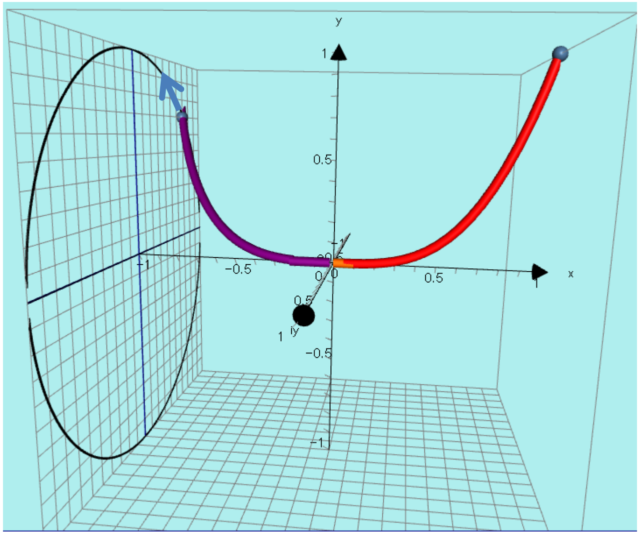
………..continuing to rotate…

The equation here is:

$$y = x^{3.25}$$



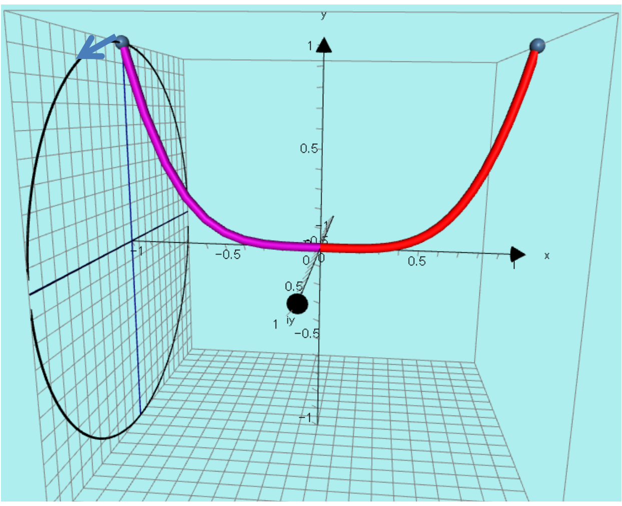
When the equation is $\mathbf{y = x^{3.5}}$ the purple section of the curve is again in a plane perpendicular to the **x, y** plane.



………..continuing to rotate…

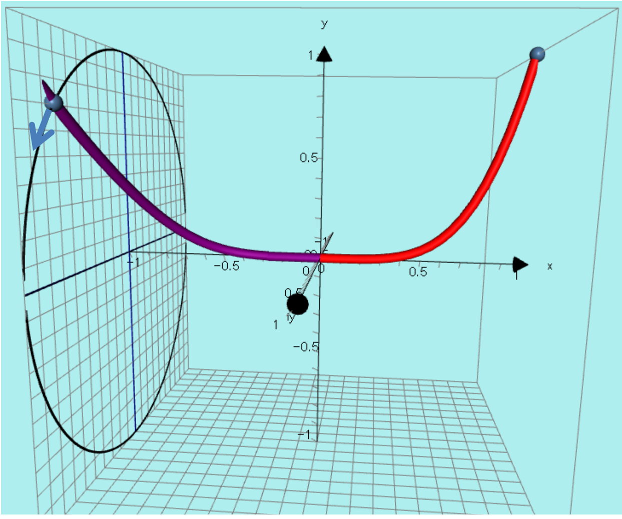
The equation here is:

$$\mathbf{y = x^{3.75}}$$



When the power is **4**, the purple curve is again back in the **x, y** plane and the equation is...

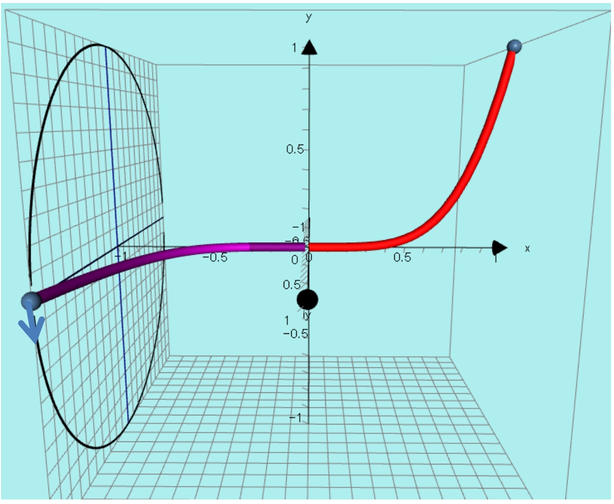
$$\mathbf{y = x^{4}}$$



……….continuing to rotate…

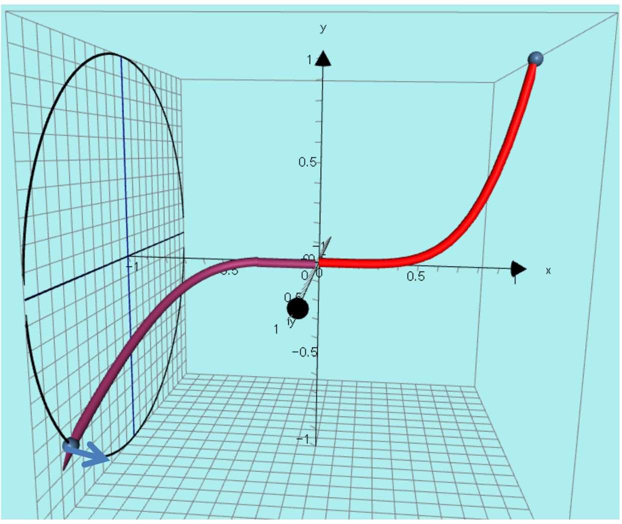
The equation here is:

$$\mathbf{y = x^{4.25}}$$



When the equation is $\mathbf{y = x^{4.5}}$ the purple section of the curve is again in a plane perpendicular to the **x, y** plane.

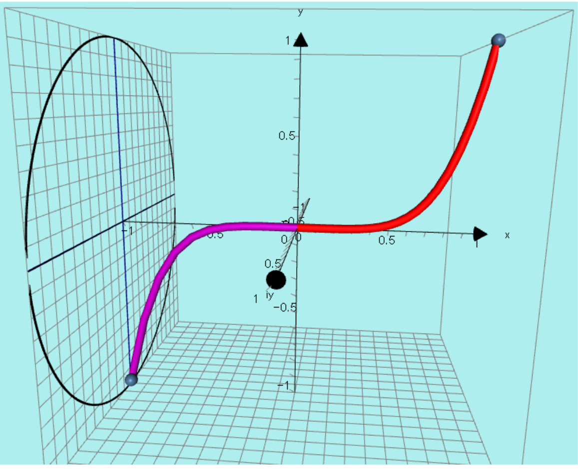
$$\mathbf{y = x^{4.5}}$$



………continuing to rotate…

The equation here is:

$$\mathbf{y = x^{4.75}}$$



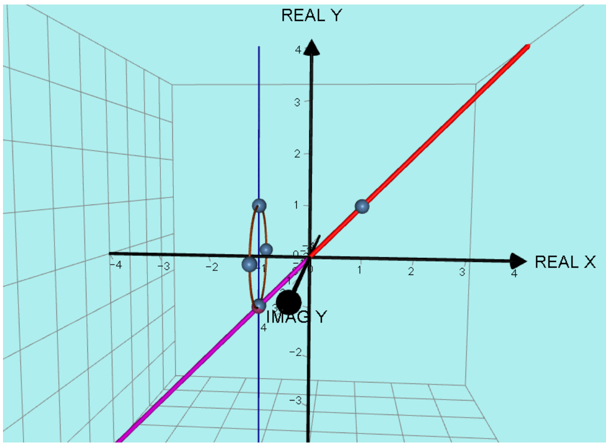
When the power is **5,** the purple curve is again back in the **x, y** plane and the equation is...

$$\mathbf{y = x^5}$$

Watch this short video of the above explanation…
https://www.screencast.com/t/tw5XrEQVfKQ

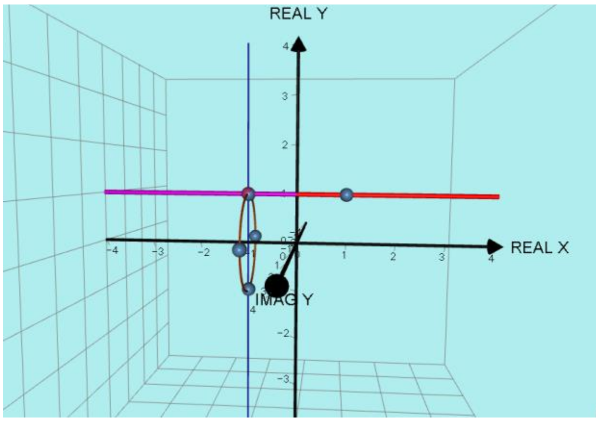
In the above theory, I started with $\mathbf{y = x^1}$ and started to **increase** the power. When I **reduced** the power there were a few surprises!

I will have a quick look at INTEGER values of **n** first.



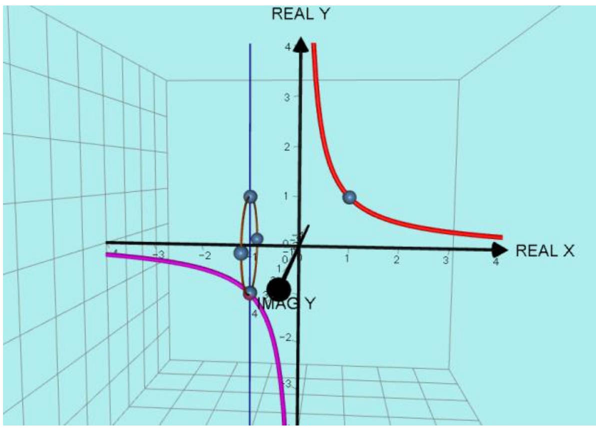
Starting again with n = 1

This is the line $\mathbf{y = x^1}$



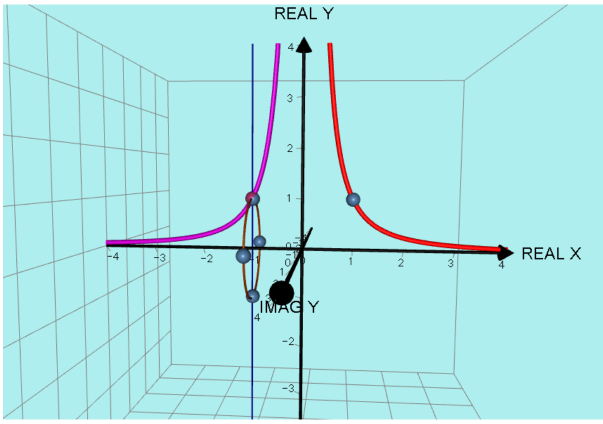
Now n = 0

This is $y = x^0$ which is the line $\mathbf{y = 1}$



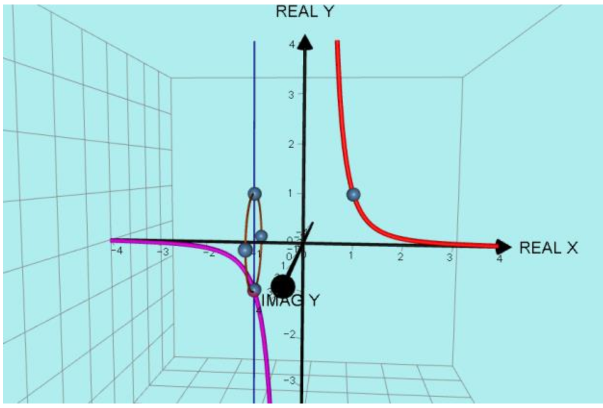
Now n = – 1

This is $\mathbf{y = x^{-1}}$ which is the well-known rectangular hyperbola.



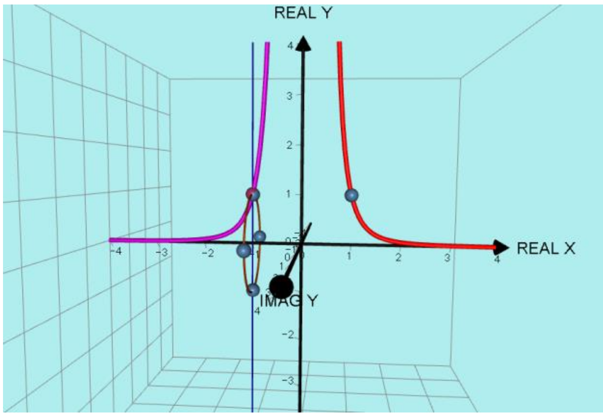
Now n = – 2

This is $\mathbf{y = x^{-2}}$



Now n = – 3

This is $\mathbf{y = x^{-3}}$

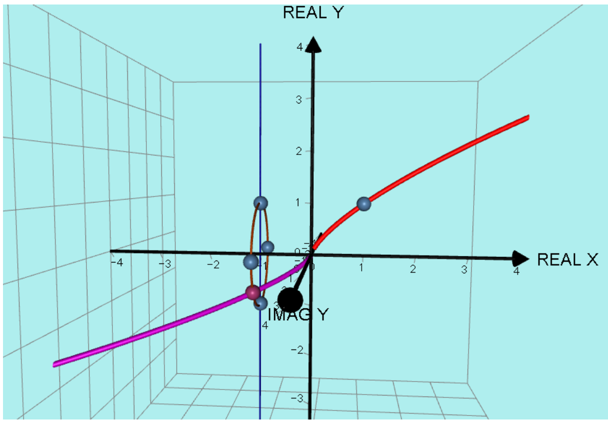


Now n = – 4

This is $\mathbf{y = x^{-4}}$

Now I will start to reduce the value of n in smaller steps.

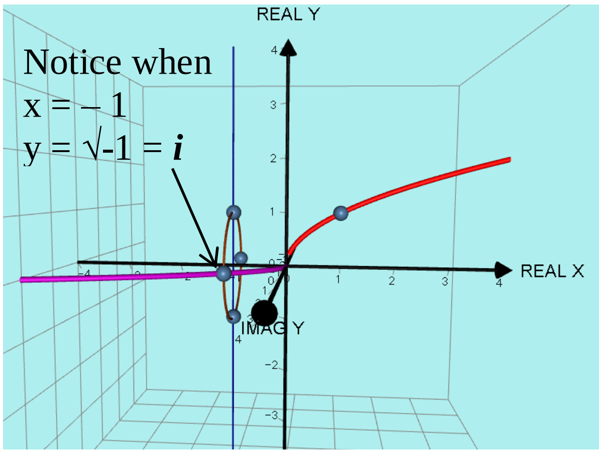
Starting from n = 1, I have reduced n to 0.7



Now n = 0.7

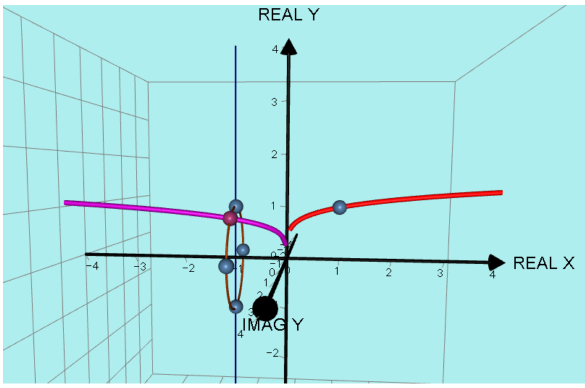
This is $\mathbf{y = x^{0.7}}$

The purple phantom has stated winding itself into the 3D space because of the complex y values.



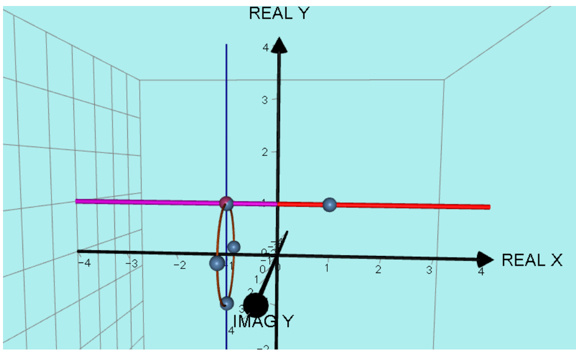
Now n = 0.5
This is $\mathbf{y = x^{0.5}} = \sqrt{x}$

This is of particular interest because it is the graph of $\mathbf{y = \sqrt{x}}$ which normally does not exist for negative x values but you can see the purple part in the complex y plane.



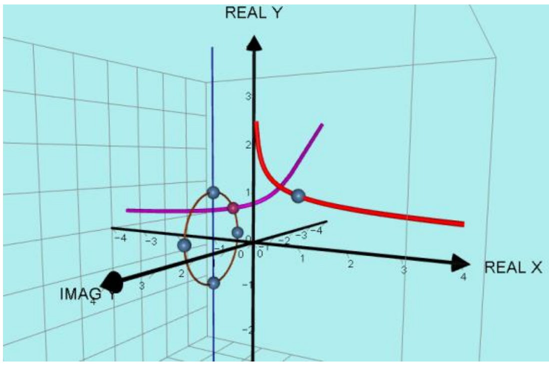
Now n = 0.2

This is $\mathbf{y = x^{0.2}}$



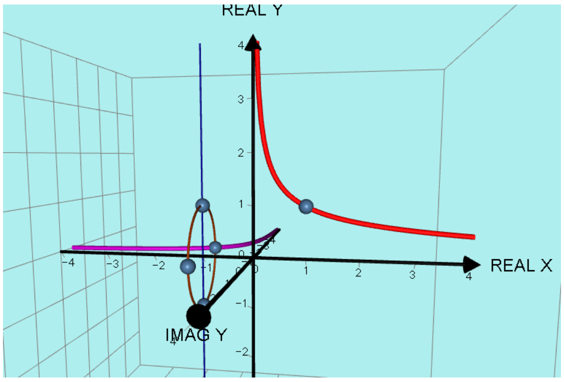
Now n = 0

This is $\mathbf{y = x^{0}} = 1$
which is a horizontal line in the x, y plane again.



Now n = – 0.3

This is $\mathbf{y = x^{-0.3}}$
I have rotated the graph for easier viewing.



Now n = – 0.5

This is $\mathbf{y = x^{-0.5}} = \frac{1}{\sqrt{x}}$
and the purple phantom is in the complex y plane at right angles to the x, y plane.

See the following video:

VIDEO FOR GRAPHS of the form **y = x^(-n)**

https://www.screencast.com/t/y5HGBxIQmSZ
